There are two basic sources of error for the system. The first is in calculation of the differential pressure ratio $\beta = \Delta P_{31}/\Delta P_{23}$. This error can be produced by errors in the analog differential pressure transducers as well as by the A/D conversions in providing differential pressure data to the computer. The second source of error is in the accuracy by which the assumed quadratic calibration curves truly represent differential pressure coefficients as functions of angle of attack. Sensitivity of system output to the first of these errors was shown to be quite low at +2.0 deg in this study but to increase drastically as angle of attack increased. Sensitivity to the second of these errors was shown to be relatively small.

Conclusions

This study has shown that a small airfoil probe, consisting of a small canard wing mounted appropriately on an airframe and properly tapped, can serve as a viable alternative as a probe for angle-of-attack sensing on aircraft. An NACA 0012 airfoil section was used in wind tunnel tests in this study, and differential pressure coefficients greater than 3.0 at high angles of attack were achieved. These coefficients are an improvement by a factor of 2.0-3.0 over comparable coefficients obtained from hemispherical probes.

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Minimum Induced Drag of Wings with Curved Planform

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Nomenclature

a	= lift slope of wing section
A_n	= *nth* coefficient of Fourier decomposition of spanwise distribution of circulation
Æ	= aspect ratio
b	= wing semispan
c	= wing chord
f	= lifting line shape function
K_I, K_T	= defined by Eq. (3)
V	= freestream velocity
W_I, W_T	= self-induced and wake-induced downwash velocities, respectively
x,y	= coordinates in the streamwise and spanwise directions, respectively
α	= angle of attack of wing cross section
Γ	= sectional circulation on the wing divided by the freestream velocity and wing semispan
ϵ	= coefficient of parabolic lifting line defined by $f = \epsilon y^2$
θ	= trigonometric transform of spanwise coordinate, $\theta = -\cos^{-1} y$
ξ,η	= dummy coordinates in x,y directions, respectively

Introduction

ONE of the classical results of incompressible aerodynamics is that the minimum induced drag on a thin flat wing of given lift is obtained when the downwash is constant.[1] This results in the well-known elliptical distribution of chord length and vorticity for straight wings.

This result has since been generalized in various directions. Kuchemann[2] showed that minimal induced drag for swept wings is obtained for a planform wider at the root and narrower at the tips than the elliptical shape. Other workers analyzed cases of wings of given lift distribution,[3] root moment, and given structural weight[4] for swept, straight planforms.

In the present Note, the problem of minimizing induced drag for given lift in the case of wings with a curved center line (line of sectional centers of pressure), including both forward and backward sweep (see Fig. 1), is examined within the framework of lifting-line theory.

Analysis

The analysis is obtained for planforms of aspect ratio Æ$\gg 1$, radius of curvature 0(Æ), and only slight 0(Æ$^{-1}$) deviations from planar wings. The derivation of this theory was described in detail in Ref. 5, so that only the relevant highlights are shown here.

Classical lifting-line theory is based on the assumption that $(x-\xi)^2 \ll (y-\eta)^2$ almost everywhere, as Æ$\gg 1$. This leads to $\eta/\xi = 0$(Æ) almost everywhere. This approach is generalized to curved wings by allowing $\xi = f(\eta) + 0(c)$ as the radius of curvature is large everywhere. The requirement of a finite radius of curvature leads to a self-induced component of downwash on the lifting line, which did not exist in straight lifting-line theory.[1-3] An integral equation[5] is obtained for the curved lifting line, correct to first order in Æ

$$\int_{-b}^{b} \frac{d\Gamma}{d\eta}\frac{1}{y-\eta}\left(1+\frac{x-f}{(x-f)^2+(y-\eta)^2}\right)d\eta + \int_{-b}^{b}\Gamma(\eta)\frac{x-f'y+f'\eta-f}{[(x-f)^2+(y-\eta)^2]^{3/2}}\,d\eta = -4\pi\alpha(y) \qquad (1)$$

The second term on the left-hand side of Eq. (1) is the new term describing the self-induced velocity. This is a singular integral equation, but the singularity which is obtained for $y=\eta$ is removed by taking a finite core radius for the vortex. This well-known technique[7] results in a new unknown, the core radius. This is obtained by calculating the induced drag twice, once in the Trefftz plane far downstream, and independently on the wing. Both results are obtained as a function of the yet unknown core radius, so that equating them gives the core size. The spanwise distribution of circulation can be written in terms of a Fourier series

$$\Gamma(\theta) = 4\sum_{n=1}^{\infty} A_n \sin(n\theta)$$

Equation (1) is then[5]

$$\sum_{n=1}^{\infty} \{8\sin n\theta + a_\infty(\theta)c(\theta)[K_I(n,\theta)\cos\alpha + nK_T(n,\theta)]\}A_n = a(\theta)c(\theta)\alpha(\theta) \qquad (2)$$

and

$$W_I = \sum_{n=1}^{\infty} A_n K_I(n,\theta) \quad \text{and} \quad W_T = \sum_{n=1}^{\infty} nA_n K_T(n,\theta) \qquad (3)$$

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The case of minimal induced drag for given lift is obtained by taking $A_n = 0$ for all $n > 1$ (Ref. 6). Substituting these conditions in Eq. (1), an explicit equation for the chord distribution of minimum induced drag is obtained

$$c(\theta) = \frac{8}{a} \frac{\sin\theta}{(\pi A\!\!R)/a + 1 - W_L \cos\alpha - W_T} \tag{4}$$

where W_L and W_T are now $A_1 K_L$ and $A_1 K_T$, respectively, from Eqs. (2) and (3). When the local induced velocity is directed downward (downwash), the chord length of a planar wing is increased, while upwash tends to decrease it. This equation can also be used to find the distribution of wing-twist for a given planform resulting in minimal drag, by solving for α. The latter is reminiscent of the analysis of twist to preserve a required lift distribution for straight and swept wings.[3]

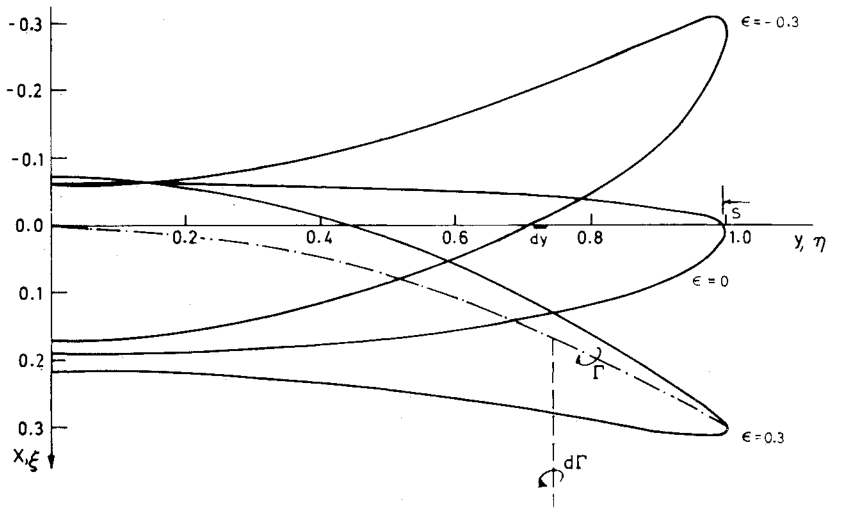

Fig. 1 Coordinate system, and planforms of minimum induced drag for given lift, with parabolically curved lifting lines. Scale is such that the semispan b defines $y = 1$. dΓ the vorticity shed over a distance $dy(d\eta)$. The planforms shown are results of the present calculations with aspect ratio 10. Note the widening of the tip region in swept-forward wings, ($\epsilon = -0.3$) relative to the elliptical wing ($\epsilon = 0$).

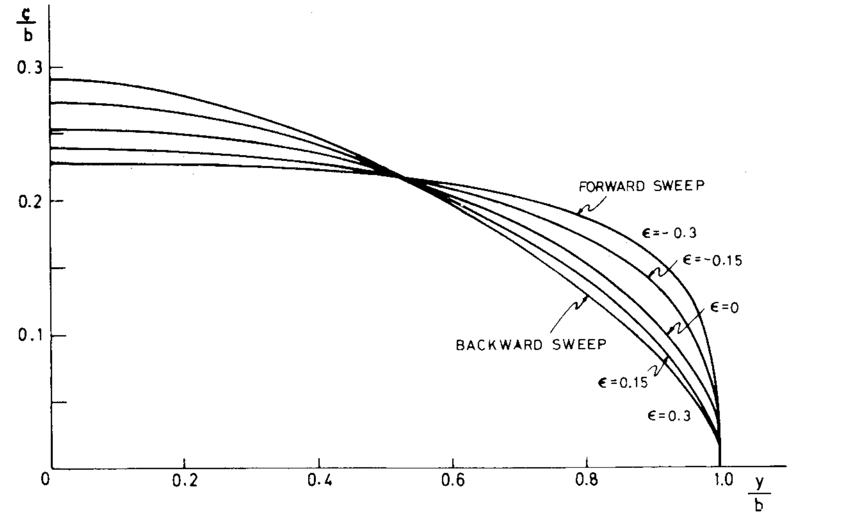

Fig. 2 Distribution of chords for planforms of minimum drag with parabolic lifting lines and aspect ratio 10. $\epsilon = 0$ is the classical elliptical distribution of chords.

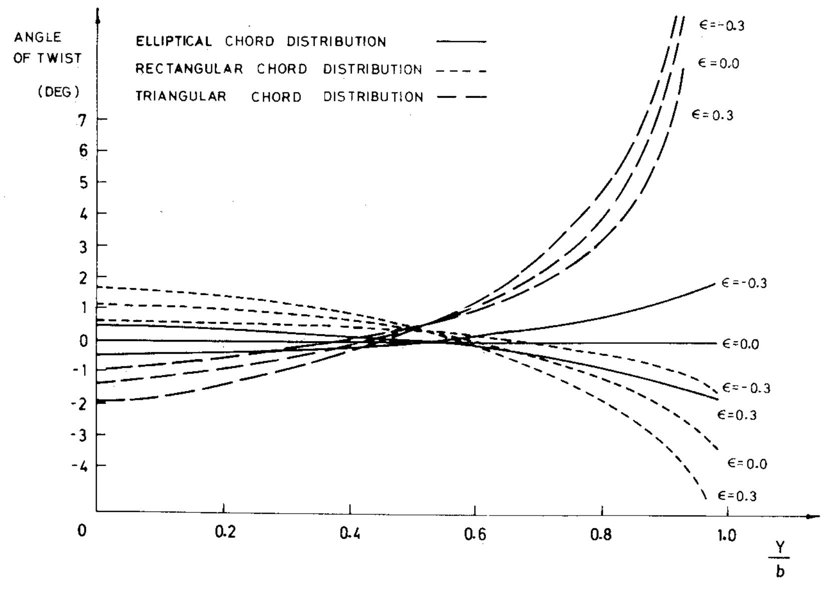

Fig. 3 Twist of wings of curved planform to obtain uniform downwash, and minimal drag.

Results

For quantitative results, a parabolic lifting line is used with sweep forward or back. This is in present nomenclature $f=\epsilon y^2$, with $-0.3<\epsilon<0.3$ to keep within the requirement of large radii of curvature, and $-1<y<1$, with $AR=10$ for all cases. The lifting line is assumed to coincide with the quarter-chord line. Figure 1 shows a top view of the planforms obtained for $\epsilon=0$ (elliptical), $\epsilon=0.3$ (swept back), and $\epsilon=-0.3$ (swept forward). The root chord is increased for swept back wings and decreased for shapes with forward sweep. More detailed distributions appear in Fig. 2. These results are in qualitative agreement with Kuchemann's calculations for straight-swept wings, the effect of curvature being to enhance the changes due to sweep. Recently, Lobert[4] showed that sweeping a tapered wing forward produces an induced drag which is only slightly higher than the optimum, while sweeping it back results in higher drag. This, again, is in agreement with present results, showing that the optimum shape (Fig. 2) for swept-back wings is more triangular. The optimum shape for swept-forward planforms is more rectangular than for wings of zero sweep.

Next, examine the related task of minimizing drag for a given chord distribution by means of controlling the angle of twist. Again take wings with a parabolic lifting line, and choose elliptical, rectangular, and triangular chord distributions. These are

$$c=\frac{8}{\pi AR}(1-y^2)^{1/2}\ \text{(elliptical)}\qquad c=\frac{2}{AR}\ \text{(rectangular)}$$

$$c=\frac{4}{AR}(1-|y|)^{1/2}\ \text{(tapered)}\tag{5}$$

Results for wings of aspect ratio 10 appear in Fig. 3. The angle of attack of the untwisted wing is 5 deg. The optimal distribution of angle of attack is shown for straight, curved-forward, and rearward planforms of each family. The drag of these planforms is slightly higher; however, than that of the elliptical wing and the shapes shown in Figs. 1 and 2.

Starting with the elliptical planforms, it may be seen that the effects of sweep-back have to be compensated by an increase in local angle of attack at the root with gradual decrease towards the tips (washout). The straight wing has uniform angle of attack (no geometrical twist) as it is already optimal, and forward-curved planforms have an angle of twist gradually increasing towards the tip (washin). The triangular planforms all have a strong washin, with this effect being much more pronounced for forward-curved wings where a total difference of 13 deg in twist is required to bring about uniform (spanwise) induced velocity. Rectangular wings exhibit an opposite behavior, with washout required in all cases. The dependence on direction of curvature is similar to the previously discussed families, in that the angle of twist is more positive (in the sense of increasing angle of attack) for forward-curved planforms.

Concluding Remarks

The results of this study show that induced drag equal to the minimum value obtained with the classical straight wing with elliptical chord distribution is achievable with planforms of both forward and rearward sweep and curvature (Fig. 1). When the chord distribution is predefined, a judicious choice of sweep combined with twist can also reduce the induced drag significantly.

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